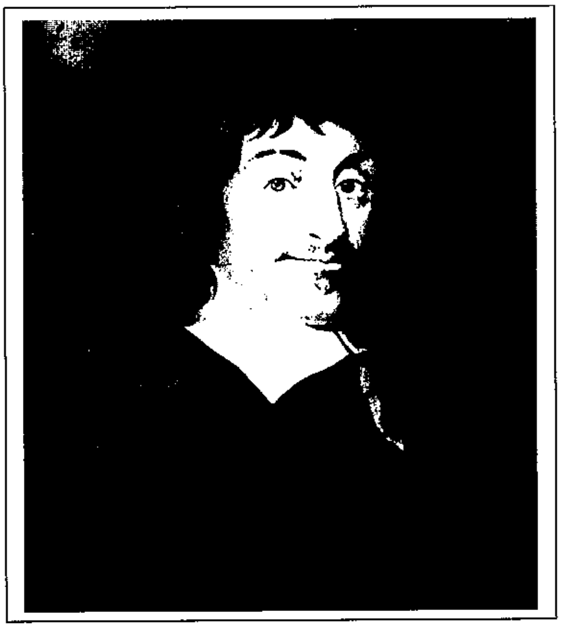
# 7 GRAPHS



**Rene Descartes** (1596–1650) was one of the greatest philosophers of his time. Strangely his restless mind only found peace and quiet as a soldier and he apparently discovered the idea of 'cartesian' geometry in a dream before the battle of Prague. The word 'cartesian' is derived from his name and his work formed the link between geometry and algebra which inevitably led to the discovery of calculus. He finally settled in Holland for ten years, but later moved to Sweden where he soon died of pneumonia.

**17** Apply rate of change to distance–time and speed–time graphs

**18** Construct tables of values and draw graphs for functions of the form $ax^n$; estimate gradients of curves by drawing tangents

**19** Interpret and obtain the equation of a straight-line graph in the form $y = mx + c$; calculate the gradient of a straight line from the coordinates of two points on it

## 7.1 Drawing accurate graphs

### Example

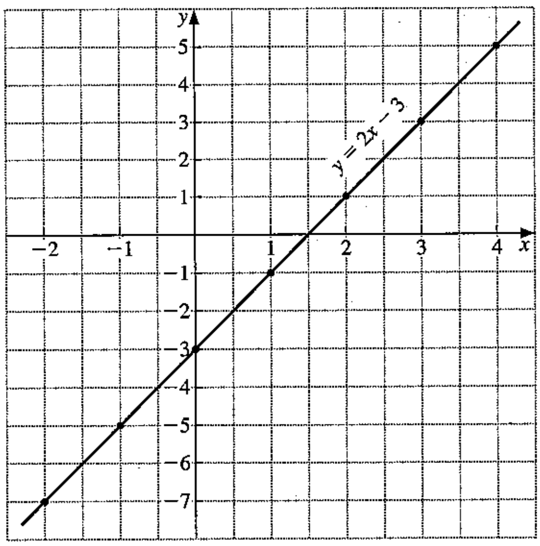
Draw the graph of $y = 2x - 3$ for values of $x$ from $-2$ to $+4$.

(a) The coordinates of points on the line are calculated in a table.

| $x$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| $2x$ | $-4$ | $-2$ | 0 | 2 | 4 | 6 | 8 |
| $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ |
| $y$ | $-7$ | $-5$ | $-3$ | $-1$ | 1 | 3 | 5 |

(b) Draw and label axes using suitable scales.

(c) Plot the points and draw a pencil line through them. Label the line with its equation.



## *Exercise 1*

Draw the following graphs, using a scale of 2 cm to 1 unit on the $x$-axis and 1 cm to 1 unit on the $y$-axis.

**1.** $y = 2x + 1$ for $-3 \leqslant x \leqslant 3$

**2.** $y = 3x - 4$ for $-3 \leqslant x \leqslant 3$

**3.** $y = 2x - 1$ for $-3 \leqslant x \leqslant 3$

**4.** $y = 8 - x$ for $-2 \leqslant x \leqslant 4$

**5.** $y = 10 - 2x$ for $-2 \leqslant x \leqslant 4$

**6.** $y = \dfrac{x+5}{2}$ for $-3 \leqslant x \leqslant 3$

**7.** $y = 3(x - 2)$ for $-3 \leqslant x \leqslant 3$

**8.** $y = \frac{1}{2}x + 4$ for $-3 \leqslant x \leqslant 3$

**9.** $v = 2t - 3$ for $-2 \leqslant t \leqslant 4$

**10.** $z = 12 - 3t$ for $-2 \leqslant t \leqslant 4$

In each question from **11** to **16**, draw the graphs on the same page and hence find the coordinates of the vertices of the polygon formed. Give the answers as accurately as your graph will allow.

**11.** (a) $y = x$ (b) $y = 8 - 4x$ (c) $y = 4x$
Take $-1 \leqslant x \leqslant 3$ and $-4 \leqslant y \leqslant 14$.

**12.** (a) $y = 2x + 1$ (b) $y = 4x - 8$ (c) $y = 1$
Take $0 \leqslant x \leqslant 5$ and $-8 \leqslant y \leqslant 12$.

**13.** (a) $y = 3x$ (b) $y = 5 - x$ (c) $y = x - 4$
Take $-2 \leqslant x \leqslant 5$ and $-9 \leqslant y \leqslant 8$.

**14.** (a) $y = -x$ (b) $y = 3x + 6$ (c) $y = 8$ (d) $x = 3\frac{1}{2}$
Take $-2 \leqslant x \leqslant 5$ and $-6 \leqslant y \leqslant 10$.

**15.** (a) $y = \frac{1}{2}(x - 8)$ (b) $2x + y = 6$ (c) $y = 4(x + 1)$
Take $-3 \leqslant x \leqslant 4$ and $-7 \leqslant y \leqslant 7$

**16.** (a) $y = 2x + 7$ (b) $3x + y = 10$ (c) $y = x$ (d) $2y + x = 4$
Take $-2 \leqslant x \leqslant 4$ and $0 \leqslant y \leqslant 13$.

**17.** The equation connecting the annual distance travelled $M$ km, of a certain car and the
annual running cost, \$$C$ is $C = \dfrac{M}{20} + 200$.
Draw the graph for $0 \leqslant M \leqslant 10\,000$ using scales of 1 cm for 1000 km for $M$ and 2 cm for \$100 for $C$.
From the graph find:
(a) the cost when the annual distance travelled is 7200 km,
(b) the annual mileage corresponding to a cost of \$320.

**18.** The equation relating the cooking time $t$ hours and the weight $w$ kg for a joint of meat
is $t = \dfrac{3w+1}{4}$.
Draw the graph for $0 \leqslant w \leqslant 5$. From the graph find:
(a) the weight of a joint requiring a cooking time of 2·8 hours,
(b) the cooking time for a joint of weight 4·1 kg.

**19.** Some drivers try to estimate their annual cost of repairs \$$c$ in relation to their average speed of driving $s$ km/h using the equation $c = 6s + 50$. Draw the graph for $0 \leqslant s \leqslant 160$. From the graph find:
(a) the estimated repair bill for a man who drives at an average speed of 65 km/h,
(b) the average speed at which a motorist drives if his annual repair bill is \$300,
(c) the annual saving for a man who, on returning from a holiday, reduces his average speed of driving from 100 km/h to 65 km/h.

**20.** The value of a car \$$v$ is related to the number of km $n$ which it has travelled by the equation

$$v = 4500 - \frac{n}{20}.$$

Draw the graph for $0 \leqslant n \leqslant 90\,000$. From the graph find:
(a) the value of a car which has travelled 3700 km,
(b) the number of km travelled by a car valued at \$3200.

## 7.2 Gradients

The gradient of a straight line is a measure of how steep it is.

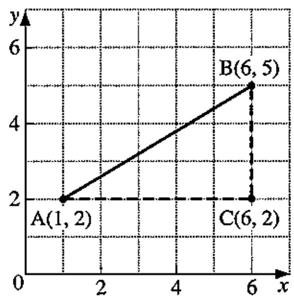
### Example 1

Find the gradient of the line joining the points A (1, 2) and B (6, 5).

$$\text{gradient of AB} = \frac{BC}{AC} = \frac{3}{5}$$

It is possible to use the formula

$$\text{gradient} = \frac{\text{difference in } y\text{-coordinates}}{\text{difference in } x\text{-coordinates}}.$$

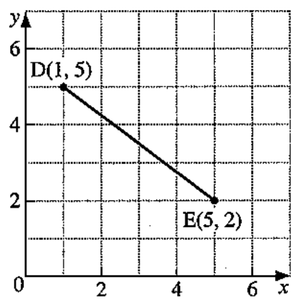



### Example 2

Find the gradient of the line joining the points D (1, 5) and E (5, 2).

$$\text{gradient of DE} = \frac{5-2}{1-5} = \frac{3}{-4} = -\frac{3}{4}$$

Note:
(a) Lines which slope upward to the right have a *positive* gradient.
(b) Lines which slope downward to the right have a *negative* gradient.



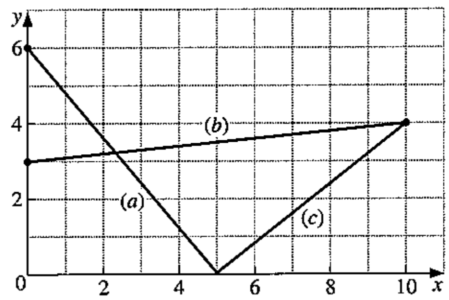
## *Exercise 2*

Calculate the gradient of the line joining the following pairs of points.

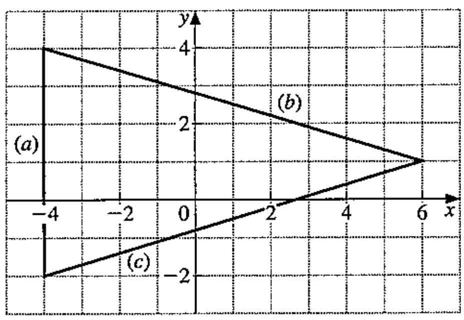
**1.** (3, 1)(5, 4)
**2.** (1, 1)(3, 5)
**3.** (3, 0)(4, 3)
**4.** (−1, 3)(1, 6)
**5.** (−2, −1)(0, 0)
**6.** (7, 5)(1, 6)
**7.** (2, −3)(1, 4)
**8.** (0, −2)(−2, 0)
**9.** $(\frac{1}{2}, 1)(\frac{3}{4}, 2)$
**10.** $(-\frac{1}{2}, 1)(0, -1)$
**11.** (3·1, 2)(3·2, 2·5)
**12.** (−7, 10)(0, 0)
**13.** $(\frac{1}{3}, 1)(\frac{1}{2}, 2)$
**14.** (3, 4)(−2, 4)
**15.** (2, 5)(1·3, 5)
**16.** (2, 3)(2, 7)
**17.** (−1, 4)(−1, 7.2)
**18.** (2·3, −2·2)(1·8, 1·8)
**19.** (0·75, 0)(0·375, −2)
**20.** (17·6, 1) (1·4, 1)
**21.** $(a, b)(c, d)$
**22.** $(m, n)(a, -b)$
**23.** $(2a, f)(a, -f)$
**24.** $(2k, -k)(k, 3k)$
**25.** $(m, 3n)(-3m, 3n)$
**26.** $\left(\frac{c}{2}, -d\right)\left(\frac{c}{4}, \frac{d}{2}\right)$

In questions **27** and **28**, find the gradient of each straight line.

**27.**




**28.**




**29.** Find the value of $a$ if the line joining the points $(3a, 4)$ and $(a, -3)$ has a gradient of 1.

**30.** (a) Write down the gradient of the line joining the points $(2m, n)$ and $(3, -4)$,
(b) Find the value of $n$ if the line is parallel to the $x$-axis,
(c) Find the value of $m$ if the line is parallel to the $y$-axis.

## 7.3 The form $y = mx + c$

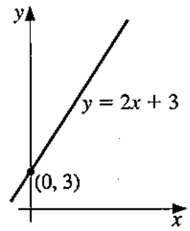
When the equation of a straight line is written in the form $y = mx + c$, the gradient of the line is $m$ and the intercept on the $y$-axis is $c$.

### Example 1

Draw the line $y = 2x + 3$ on a *sketch* graph.

The word 'sketch' implies that we do not plot a series of points but simply show the position and slope of the line.

The line $y = 2x + 3$ has a gradient of 2 and cuts the $y$-axis at (0, 3).



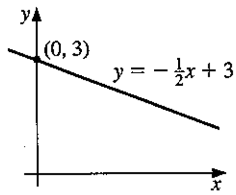

### Example 2

Draw the line $x + 2y - 6 = 0$ on a sketch graph.

(a) Rearrange the equation to make $y$ the subject.

$$x + 2y - 6 = 0$$
$$2y = -x + 6$$
$$y = -\tfrac{1}{2}x + 3$$

(b) The line has a gradient of $-\frac{1}{2}$ and cuts the $y$-axis at (0, 3).




### *Exercise 3*

In questions **1** to **20**, find the gradient of the line and the intercept on the $y$-axis. Hence draw a small sketch graph of each line.

| | | | |
|---|---|---|---|
| **1.** $y = x + 3$ | **2.** $y = x - 2$ | **3.** $y = 2x + 1$ | **4.** $y = 2x - 5$ |
| **5.** $y = 3x + 4$ | **6.** $y = \frac{1}{2}x + 6$ | **7.** $y = 3x - 2$ | **8.** $y = 2x$ |
| **9.** $y = \frac{1}{4}x - 4$ | **10.** $y = -x + 3$ | **11.** $y = 6 - 2x$ | **12.** $y = 2 - x$ |
| **13.** $y + 2x = 3$ | **14.** $3x + y + 4 = 0$ | **15.** $2y - x = 6$ | **16.** $3y + x - 9 = 0$ |
| **17.** $4x - y = 5$ | **18.** $3x - 2y = 8$ | **19.** $10x - y = 0$ | **20.** $y - 4 = 0$ |

## Finding the equation of a line

### Example

Find the equation of the straight line which passes through (1, 3) and (3, 7).

(a) Let the equation of the line take the form $y = mx + c$.

The gradient, $m = \dfrac{7-3}{3-1} = 2$

so we may write the equation as

$$y = 2x + c \qquad \ldots[1]$$

(b) Since the line passes through (1, 3), substitute 3 for $y$ and 1 for $x$ in [1].

$$\therefore\quad 3 = 2 \times 1 + c$$
$$1 = c$$

The equation of the line is $y = 2x + 1$.

### Exercise 4

In questions **1** to **11** find the equation of the line which:

**1.** Passes through (0, 7) at a gradient of 3

**2.** Passes through (0, −9) at a gradient of 2

**3.** Passes through (0, 5) at a gradient of −1

**4.** Passes through (2, 3) at a gradient of 2

**5.** Passes through (2, 11) at a gradient of 3

**6.** Passes through (4, 3) at a gradient of −1

**7.** Passes through (6, 0) at a gradient of $\frac{1}{2}$

**8.** Passes through (2, 1) and (4, 5)

**9.** Passes through (5, 4) and (6, 7)

**10.** Passes through (0, 5) and (3, 2)

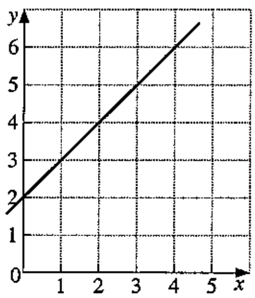
**11.** Passes through (3, −3) and (9, −1)

## 7.4 Linear laws

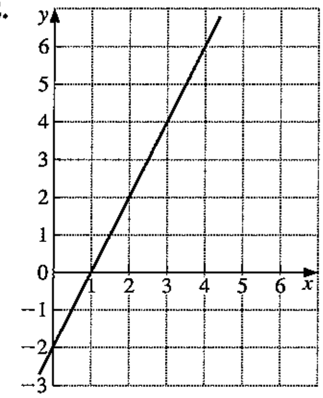
### Exercise 5

In Questions **1** to **3** find the equation of the line in the form $y = \ldots\ldots$ .

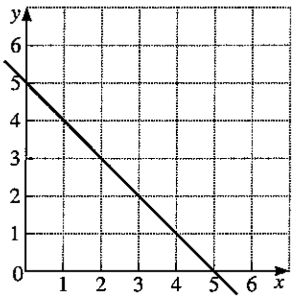
**1.**




**2.**




**3.**




**4.** In an experiment, the following measurements of the variables $q$ and $t$ were taken:

| | | | | | | |
|---|---|---|---|---|---|---|
| $q$ | 0·5 | 1·0 | 1·5 | 2·0 | 2·5 | 3·0 |
| $t$ | 3·85 | 5·0 | 6·1 | 7·0 | 7·75 | 9·1 |

A scientist suspects that $q$ and $t$ are related by an equation of the form $t = mq + c$, ($m$ and $c$ constants). Plot the values obtained from the experiment and draw the line of best fit through the points. Plot $q$ on the horizontal axis with a scale of 4 cm to 1 unit, and $t$ on the vertical axis with a scale of 2 cm to 1 unit. Find the gradient and intercept on the $t$-axis and hence estimate the values of $m$ and $c$.

5. In an experiment, the following measurements of $p$ and $z$ were taken:

| $z$ | 1·2 | 2·0 | 2·4 | 3·2 | 3·8 | 4·6 |
|---|---|---|---|---|---|---|
| $p$ | 11·5 | 10·2 | 8·8 | 7·0 | 6 | 3·5 |

Plot the points on a graph with $z$ on the horizontal axis and draw the line of best fit through the points. Hence estimate the values of $n$ and $k$ if the equation relating $p$ and $z$ is of the form $p = nz + k$.

6. In an experiment the following measurements of $t$ and $z$ were taken:

| $t$ | 1·41 | 2·12 | 2·55 | 3·0 | 3·39 | 3·74 |
|---|---|---|---|---|---|---|
| $z$ | 3·4 | 3·85 | 4·35 | 4·8 | 5·3 | 5·75 |

Draw a graph, plotting $t^2$ on the horizontal axis and $z$ on the vertical axis, and hence confirm that the equation connecting $t$ and $z$ is of the form $z = mt^2 + c$. Find approximate values for $m$ and $c$.

## 7.5 Plotting curves

### Example

Draw the graph of the function $y = 2x^2 + x - 6$, for $-3 \leqslant x \leqslant 3$.

(a)

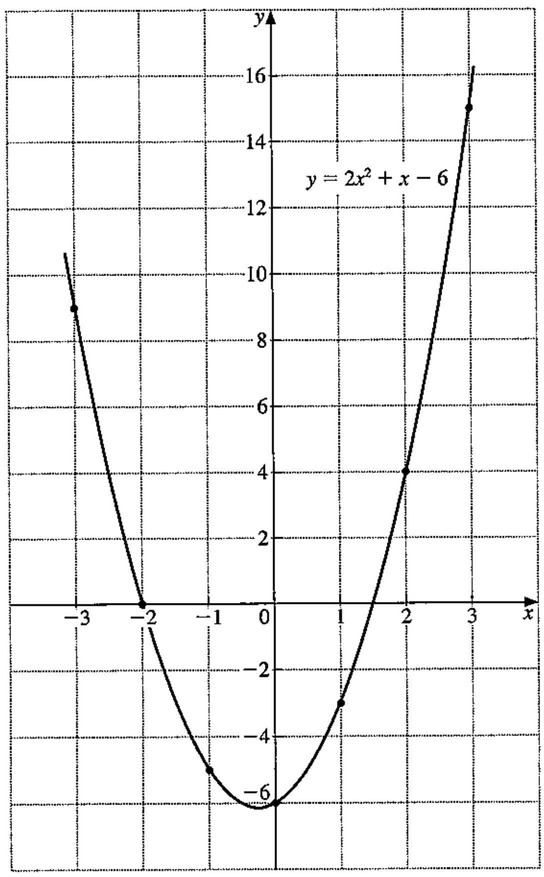
| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $2x^2$ | 18 | 8 | 2 | 0 | 2 | 8 | 18 |
| $x$ | $-3$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 |
| $-6$ | $-6$ | $-6$ | $-6$ | $-6$ | $-6$ | $-6$ | $-6$ |
| $y$ | 9 | 0 | $-5$ | $-6$ | $-3$ | $-4$ | 15 |

(b) Draw and label axes using suitable scales.

(c) Plot the points and draw a smooth curve through them with a pencil.

(d) Check any points which interrupt the smoothness of the curve.

(e) Label the curve with its equation.



Sometimes the function notation $f(x)$ is used. $f(x)$ means 'a function of $x$'.
If, for example, $f(x) = x^2 + 2x$ then the graph of $y = f(x)$ is simply the graph of $y = x^2 + 2x$.

Functions are explained in greater detail on page 265.

An alternative way of writing $f(x) = x^2 + 2x$ is $f: x \to x^2 + 2x$

To find the value of $y$ when $x = 1$ we obtain,

$f(1) = 1^2 + 2 \times 1 = 3$, so $y = 3$

Similarly,

$f(2) = 2^2 + 2 \times 2 = 8$

$f(3) = 3^2 + 2 \times 3 = 15$, and so on.

### *Exercise 6*

Draw the graphs of the following functions using a scale of 2 cm for 1 unit on the $x$-axis and 1 cm for 1 unit on the $y$-axis.

1. $y = x^2 + 2x$, for $-3 \leqslant x \leqslant 3$
2. $y = x^2 + 4x$, for $-3 \leqslant x \leqslant 3$
3. $y = x^2 - 3x$, for $-3 \leqslant x \leqslant 3$
4. $y = x^2 + 2$, for $-3 \leqslant x \leqslant 3$
5. $y = x^2 - 7$, for $-3 \leqslant x \leqslant 3$
6. $y = x^2 + x - 2$, for $-3 \leqslant x \leqslant 3$
7. $y = x^2 + 3x - 9$, for $-4 \leqslant x \leqslant 3$
8. $y = x^2 - 3x - 4$, for $-2 \leqslant x \leqslant 4$
9. $y = x^2 - 5x + 7$, for $0 \leqslant x \leqslant 6$
10. $y = 2x^2 - 6x$, for $-1 \leqslant x \leqslant 5$
11. $y = 2x^2 + 3x - 6$, for $-4 \leqslant x \leqslant 2$
12. $y = 3x^2 - 6x + 5$, for $-1 \leqslant x \leqslant 3$
13. $y = 2 + x - x^2$, for $-3 \leqslant x \leqslant 3$
14. $f(x) = 1 - 3x - x^2$, for $-5 \leqslant x \leqslant 2$
15. $f(x) = 3 + 3x - x^2$, for $-2 \leqslant x \leqslant 5$
16. $f(x) = 7 - 3x - 2x^2$, for $-3 \leqslant x \leqslant 3$
17. $f(x) = 6 + x - 2x^2$, for $-3 \leqslant x \leqslant 3$
18. $f: x \to 8 + 2x - 3x^2$, for $-2 \leqslant x \leqslant 3$
19. $f: x \to x(x - 4)$, for $-1 \leqslant x \leqslant 6$
20. $f: x \to (x + 1)(2x - 5)$, for $-3 \leqslant x \leqslant 3$.

## Example

Draw the graph of $y = \dfrac{12}{x} + x - 6$, for $1 \leqslant x \leqslant 8$.

Use the graph to find approximate values for:

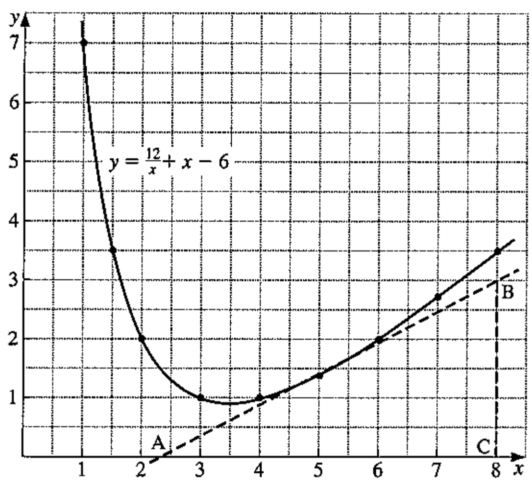
(a) the minimum value of $\dfrac{12}{x} + x - 6$

(b) the value of $\dfrac{12}{x} + x - 6$, when $x = 2{\cdot}25$

(c) the gradient of the tangent to the curve drawn at the point where $x = 5$.

Here is the table of values:

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 1·5 |
|---|---|---|---|---|---|---|---|---|---|
| $\frac{12}{x}$ | 12 | 6 | 4 | 3 | 2·4 | 2 | 1·71 | 1·5 | 8 |
| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 1·5 |
| $-6$ | $-6$ | $-6$ | $-6$ | $-6$ | $-6$ | $-6$ | $-6$ | $-6$ | $-6$ |
| $y$ | 7 | 2 | 1 | 1 | 1·4 | 2 | 2·71 | 3·5 | 3·5 |




Notice that an 'extra' value of $y$ has been calculated at $x = 1{\cdot}5$ because of the large difference between the $y$-values at $x = 1$ and $x = 2$.

(a) From the graph, the minimum value of $\frac{12}{x} + x - 6$ (i.e. $y$) is approximately 0·9.

(b) At $x = 2{\cdot}25$, $y$ is approximately 1·6.

(c) The tangent AB is drawn to touch the curve at $x = 5$

The gradient of $AB = \frac{BC}{AC}$

$$\text{gradient} = \frac{3}{8 - 2{\cdot}4} = \frac{3}{5{\cdot}6} \approx 0{\cdot}54$$

It is difficult to obtain an accurate value for the gradient of a tangent so the above result is more realistically 'approximately 0·5'.

## *Exercise 7*

Draw the following curves. The scales given are for one unit of $x$ and $y$.

**1.** $y = x^2$, for $0 \leqslant x \leqslant 6$.
(Scales: 2 cm for $x$, $\frac{1}{2}$ cm for $y$)
Find:
(a) the gradient of the tangent to the curve at $x = 2$,
(b) the gradient of the tangent to the curve at $x = 4$,
(c) the $y$-value at $x = 3{\cdot}25$.

**2.** $y = x^2 - 3x$, for $-2 \leqslant x \leqslant 5$.
(Scales: 2 cm for $x$, 1 cm for $y$)
Find:
(a) the gradient of the tangent to the curve at $x = 3$,
(b) the gradient of the tangent to the curve at $x = -1$,
(c) the value of $x$ where the gradient of the curve is zero.

**3.** $y = 5 + 3x - x^2$, for $-2 \leqslant x \leqslant 5$.
(Scales: 2 cm for $x$, 1 cm for $y$)
Find:
(a) the maximum value of the function $5 + 3x - x^2$,
(b) the gradient of the tangent to the curve at $x = 2{\cdot}5$,
(c) the two values of $x$ for which $y = 2$.

**4.** $y = \frac{12}{x}$, for $1 \leqslant x \leqslant 10$.
(Scales: 1 cm for $x$ and $y$)

**5.** $y = \frac{9}{x}$, for $1 \leqslant x \leqslant 10$.
(Scales: 1 cm for $x$ and $y$)

**6.** $y = \frac{12}{x+1}$, for $0 \leqslant x \leqslant 8$.
(Scales: 2 cm for $x$, 1 cm for $y$)

**7.** $y = \frac{8}{x-4}$, for $-4 \leqslant x \leqslant 3{\cdot}5$.
(Scales: 2 cm for $x$, 1 cm for $y$)

**8.** $y = \dfrac{15}{3-x}$, for $-4 \leqslant x \leqslant 2$.
(Scales: 2 cm for $x$, 1 cm for $y$)

**9.** $y = \dfrac{x}{x+4}$, for $-3{\cdot}5 \leqslant x \leqslant 4$.
(Scales: 2 cm for $x$ and $y$)

**10.** $y = \dfrac{3x}{5-x}$, for $-3 \leqslant x \leqslant 4$.
(Scales: 2 cm for $x$, 1 cm for $y$)

**11.** $y = \dfrac{x+8}{x+1}$, for $0 \leqslant x \leqslant 8$.
(Scales: 2 cm for $x$ and $y$)

**12.** $y = \dfrac{x-3}{x+2}$, for $-1 \leqslant x \leqslant 6$.
(Scales: 2 cm for $x$ and $y$)

**13.** $y = \dfrac{10}{x} + x$, for $1 \leqslant x \leqslant 7$.
(Scales: 2 cm for $x$, 1 cm for $y$)

**14.** $y = \dfrac{12}{x} - x$, for $1 \leqslant x \leqslant 7$.
(Scales: 2 cm for $x$, 1 cm for $y$)

**15.** $y = \dfrac{15}{x} + x - 7$, for $1 \leqslant x \leqslant 7$.
(Scales: 1 cm for $x$ and $y$)
Find: (a) the minimum value of $y$,
(b) the $y$ value when $x = 5{\cdot}5$.

**16.** $y = x^3 - 2x^2$, for $0 \leqslant x \leqslant 4$.
(Scales: 2 cm for $x$, $\frac{1}{2}$ cm for $y$)
Find: (a) the $y$ value at $x = 2{\cdot}5$,
(b) the $x$ value at $y = 15$.

**17.** $y = \frac{1}{10}(x^3 + 2x + 20)$, for $-3 \leqslant x \leqslant 3$.
(Scales: 2 cm for $x$ and $y$)
Find:
(a) the $x$-value where $x^3 + 2x + 20 = 0$,
(b) the gradient of the tangent to the curve at $x = 2$.

**18.** Copy and complete the table for the function $y = 7 - 5x - 2x^2$, giving values of $y$ correct to one decimal place.

| $x$ | $-4$ | $-3{\cdot}5$ | $-3$ | $-2{\cdot}5$ | $-2$ | $-1{\cdot}5$ |
|---|---|---|---|---|---|---|
| 7 | 7 | 7 | | 7 | | 7 |
| $-5x$ | 20 | 17·5 | | 12·5 | | 7·5 |
| $-2x^2$ | $-32$ | $-24{\cdot}5$ | | $-12{\cdot}5$ | | $-4{\cdot}5$ |
| $y$ | 5 | 0 | | 7 | | 10 |

| $x$ | $-1$ | $-0{\cdot}5$ | 0 | 0·5 | 1 | 1·5 | 2 |
|---|---|---|---|---|---|---|---|
| 7 | | 7 | | 7 | | 7 | |
| $-5x$ | | 2·5 | | $-2{\cdot}5$ | | $-7{\cdot}5$ | |
| $-2x^2$ | | $-0{\cdot}5$ | | $-0{\cdot}5$ | | $-4{\cdot}5$ | |
| $y$ | | 9 | | 4 | | $-5$ | |

Draw the graph, using a scale of 2 cm for $x$ and 1 cm for $y$. Find:
(a) the gradient of the tangent to the curve at $x = -2{\cdot}5$,
(b) the maximum value of $y$,
(c) the value of $x$ at which this maximum value occurs.

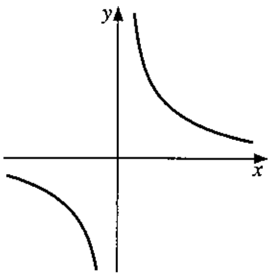
**19.** Draw the graph of $y = \dfrac{x}{x^2+1}$, for $-6 \leqslant x \leqslant 6$.
(Scales: 1 cm for $x$, 10 cm for $y$)

**20.** Draw the graph of $E = \frac{5000}{x} + 3x$ for $10 \leqslant x \leqslant 80$. (Scales: 1 cm to 5 units for $x$ and 1 cm to 25 units for $E$)
From the graph find:
(a) the minimum value of $E$,
(b) the value of $x$ corresponding to this minimum value,
(c) the range of values of $x$ for which $E$ is less than 275.

## Sketch graphs

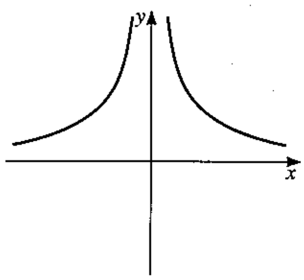
You need to recognise and be able to sketch the graphs of:

$y = \frac{a}{x}$



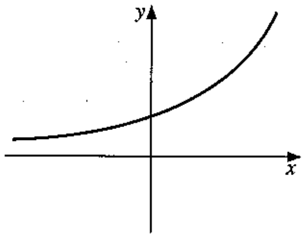

It discontinues at $x = 0$.
$x = 0$ is an asymptote.

$y = \frac{a}{x^2}$




It discontinues at $x = 0$.
$x = 0$ is an asymptote.

$y = a^x$

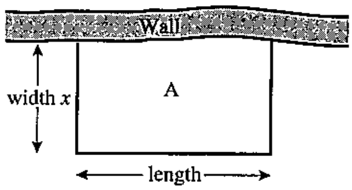



$y = a^x$ is known as an *exponential* function.

### *Exercise 8*

**1.** A rectangle has a perimeter of 14 cm and length $x$ cm. Show that the width of the rectangle is $(7 - x)$ cm and hence that the area $A$ of the rectangle is given by the formula $A = x(7 - x)$. Draw the graph, plotting $x$ on the horizontal axis with a scale of 2 cm to 1 unit, and $A$ on the vertical axis with a scale of 1 cm to 1 unit. Take $x$ from 0 to 7. From the graph find:
(a) the area of the rectangle when $x = 2{\cdot}25$ cm,
(b) the dimensions of the rectangle when its area is 9 cm$^2$,
(c) the maximum area of the rectangle,
(d) the length and width of the rectangle corresponding to the maximum area,
(e) what shape of rectangle has the largest area.

**2.** A farmer has 60 m of wire fencing which he uses to make a rectangular pen for his sheep. He uses a stone wall as one side of the pen so the wire is used for only 3 sides of the pen.



If the width of the pen is $x$ m, what is the length (in terms of $x$)?
What is the area $A$ of the pen?
Draw a graph with area $A$ on the vertical axis and the width $x$ on the horizontal axis. Take values of $x$ from 0 to 30.
What dimensions should the pen have if the farmer wants to enclose the largest possible area?

**3.** A ball is thrown in the air so that $t$ seconds after it is thrown, its height $h$ metres above its starting point is given by the function $h = 25t - 5t^2$. Draw the graph of the function for $0 \leqslant t \leqslant 6$, plotting $t$ on the horizontal axis with a scale of 2 cm to 1 second, and $h$ on the vertical axis with a scale of 2 cm for 10 metres. Use the graph to find:
(a) the time when the ball is at its greatest height,
(b) the greatest height reached by the ball,
(c) the interval of time during which the ball is at a height of more than 30 m.

**4.** The velocity $v$ m/s of a missile $t$ seconds after launching is given by the equation $v = 54t - 2t^3$. Draw a graph, plotting $t$ on the horizontal axis with a scale of 2 cm to 1 second, and $v$ on the vertical axis with a scale of 1 cm for 10 m/s. Take values of $t$ from 0 to 5.
Use the graph to find:
(a) the maximum velocity reached,
(b) the time taken to accelerate to a velocity of 70 m/s,
(c) the interval of time during which the missile is travelling at more than 100 m/s.

**5.** Draw the graph of $y = 2^x$, for $-4 \leqslant x \leqslant 4$.
(Scales: 2 cm for $x$, 1 cm for $y$)

**6.** Draw the graph of $y = 3^x$, for $-3 \leqslant x \leqslant 3$.
(Scales: 2 cm for $x$, $\frac{1}{2}$ cm for $y$)
Find the gradient of the tangent to the curve at $x = 1$.

**7.** Consider the equation $y = \dfrac{1}{x}$.

When $x = \dfrac{1}{2}$, $y = \dfrac{1}{\frac{1}{2}} = 2$.

When $x = \frac{1}{100}$, $y = \frac{1}{\frac{1}{100}} = 100$.

As the denominator of the fraction $\frac{1}{x}$ gets smaller, the answer gets larger. An 'infinitely small' denominator gives an 'infinitely large' answer.

We write $\frac{1}{0} \to \infty$. '$\frac{1}{0}$ tends to an infinitely large number.'

Draw the graph of $y = \frac{1}{x}$ for $x = -4, -3, -2, -1, -0{\cdot}5, -0{\cdot}25, 0{\cdot}5, 1, 2, 3, 4$

(Scales: 2 cm for $x$ and $y$)

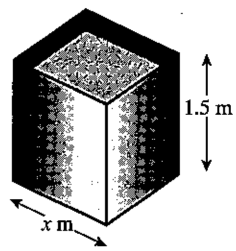
**8.** Draw the graph of $y = x + \frac{1}{x}$ for $x = -4, -3, -2, -1, -0{\cdot}5, -0{\cdot}25, 0{\cdot}25, 0{\cdot}5, 1, 2, 3, 4$

(Scales: 2 cm for $x$ and $y$)

**9.** Draw the graph of $y = x + \frac{1}{x^2}$ for $x = -4, -3, -2, -1, -0{\cdot}5, -0{\cdot}25, 0{\cdot}25, 0{\cdot}5, 1, 2, 3, 4$

(Scales: 2 cm for $x$, 1 cm for $y$)

**10.** This sketch shows a water tank with a square base. It is 1·5 m high, and the length of the base is $x$ metres.




(a) Explain why the volume of the tank is given by the formula $V = 1{\cdot}5x^2$

(b) Complete the table to show the volume for various values of $x$.

| $x$ | 0·1 | 0·2 | 0·3 | 0·4 | 0·5 | 0·6 | 0·7 | 0·8 |
|---|---|---|---|---|---|---|---|---|
| $V$ | 0·02 | 0·06 | 0·14 | 0·24 | | 0·54 | 0·74 | 0·96 |

| $x$ | 0·9 | 1·0 | 1·1 | 1·2 | 1·3 | 1·4 | 1·5 |
|---|---|---|---|---|---|---|---|
| $V$ | 1·2 | | 1·82 | | 2·54 | | 3·38 |

(c) Draw the graph of $V = 1{\cdot}5x^2$ for values of $x$ from 0 to 1·5.

(d) What value of $x$ will give a volume of $3\,\text{m}^3$?

(e) A guest house needs a tank with a volume at least $2\,\text{m}^3$. To fit the tank into the loft, it must not be more than 1·3 m wide. Write down the range of values for $x$ which will satisfy these conditions.

## 7.6 Interpreting graphs

### *Exercise 9*

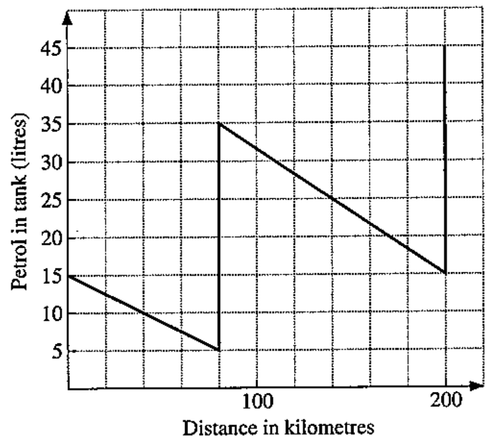
1. Kendal Motors hire out vans at a basic charge of \$35 plus a charge of 20c per km travelled. Copy and complete the table where $x$ is the number of km travelled and $C$ is the total cost in dollars.

| $x$ | 0 | 50 | 100 | 150 | 200 | 250 | 300 |
|---|---|---|---|---|---|---|---|
| $C$ | 35 | | | 65 | | | 95 |

   Draw a graph of $C$ against $x$, using scales of 2 cm for 50 km on the $x$-axis and 1 cm for \$10 on the $C$-axis.
   (a) Use the graph to find the number of miles travelled when the total cost was \$71.
   (b) What is the formula connecting $C$ and $x$?

2. A car travels along a motorway and the amount of petrol in its tank is monitored as shown on the graph below.
   (a) How much petrol was bought at the first stop?
   (b) What was the petrol consumption in km per litre:
      (i) before the first stop,
      (ii) between the two stops?
   (c) What was the average petrol consumption over the 200 km?

   After it leaves the second service station the car encounters road works and slow traffic for the next 20 km. Its petrol consumption is reduced to 4 km per litre. After that, the road clears and the car travels a further 75 km during which time the consumption is 7·5 km/litre. Draw the graph above and extend it to show the next 95 km. How much petrol is in the tank at the end of the journey?




3. A firm makes a profit of $P$ thousand dollars from producing $x$ thousand tiles.
   Corresponding values of $P$ and $x$ are given below

| $x$ | 0 | 0·5 | 1·0 | 1·5 | 2·0 | 2·5 | 3·0 |
|---|---|---|---|---|---|---|---|
| $P$ | −1·0 | 0·75 | 2·0 | 2·75 | 3·0 | 2·75 | 2·0 |

   Using a scale of 4 cm to one unit on each axis, draw the graph of $P$ against $x$. [Plot $x$ on the horizontal axis.] Use your graph to find:
   (a) the number of tiles the firm should produce in order to make the maximum profit.
   (b) the minimum number of tiles that should be produced to cover the cost of production.
   (c) the range of values of $x$ for which the profit is more than \$2850.

**4** A small firm increases its monthly expenditure on advertising and records its monthly income from sales.

| Month | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Expenditure ($) | 100 | 200 | 300 | 400 | 500 | 600 | 700 |
| Income ($) | 280 | 450 | 560 | 630 | 680 | 720 | 740 |

Draw a graph to display this information.

(a) Is it wise to spend $100 per month on advertising?

(b) Is it wise to spend $700 per month?

(c) What is the most sensible level of expenditure on advertising?

## 7.7 Graphical solution of equations

Accurately drawn graphs enable us to find approximate solutions to a wide range of equations, many of which are impossible to solve exactly by 'conventional' methods.

### Example 1

Draw the graph of the function

$$y = 2x^2 - x - 3$$

for $-2 \leqslant x \leqslant 3$. Use the graph to find approximate solutions to the following equations.

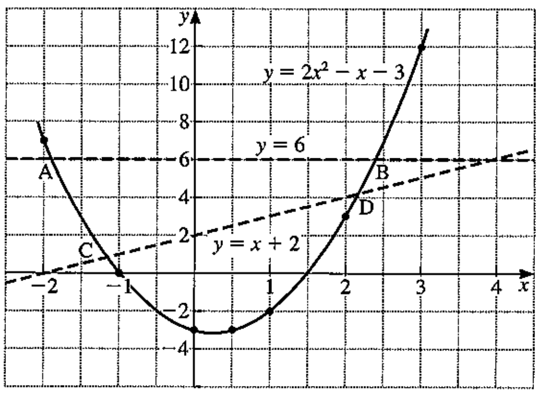
(a) $2x^2 - x - 3 = 6$

(b) $2x^2 - x = x + 5$

The table of values for $y = 2x^2 - x - 3$ is found. Note the 'extra' value at $x = \frac{1}{2}$.

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $\frac{1}{2}$ |
|---|---|---|---|---|---|---|---|
| $2x^2$ | 8 | 2 | 0 | 2 | 8 | 18 | $\frac{1}{2}$ |
| $-x$ | 2 | 1 | 0 | $-1$ | $-2$ | $-3$ | $-\frac{1}{2}$ |
| $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ | $-3$ |
| $y$ | 7 | 0 | $-3$ | $-2$ | 3 | 12 | $-3$ |

The graph drawn from this table is opposite.




(a) To solve the equation $2x^2 - x - 3 = 6$, the line $y = 6$ is drawn. At the points of intersection (A and B), $y$ simultaneously equals both 6 and $(2x^2 - x - 3)$.
So we may write

$$2x^2 - x - 3 = 6$$

The solutions are the $x$-values of the points A and B.
i.e. $x = -1{\cdot}9$ and $x = 2{\cdot}4$ approx.

(b) To solve the equation $2x^2 - x = x + 5$, we rearrange the equation to obtain the function $(2x^2 - x - 3)$ on the left-hand side. In this case, subtract 3 from both sides.

$$2x^2 - x - 3 = x + 5 - 3$$
$$2x^2 - x - 3 = x + 2$$

If we now draw the line $y = x + 2$, the solutions of the equation are given by the $x$-values of C and D, the points of intersection. i.e. $x = -1{\cdot}2$ and $x = 2{\cdot}2$ approx.

It is important to rearrange the equation to be solved so that the function already plotted is on one side.

## Example 2

Assuming that the graph of $y = x^2 - 3x + 1$ has been drawn, find the equation of the line which should be drawn to solve the equation:

$$x^2 - 4x + 3 = 0$$

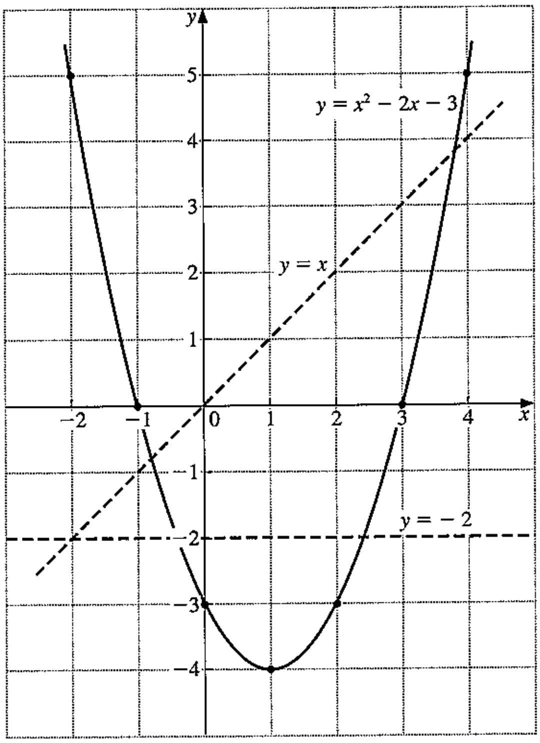
Rearrange $x^2 - 4x + 3 = 0$ in order to obtain $(x^2 - 3x + 1)$ on the left-hand side.

| | |
|---|---|
| | $x^2 - 4x + 3 = 0$ |
| add $x$ | $x^2 - 3x + 3 = x$ |
| subtract 2 | $x^2 - 3x + 1 = x - 2$ |

Therefore draw the line $y = x - 2$ to solve the equation.

## *Exercise 10*

1. In the diagram, the graph of $y = x^2 - 2x - 3$, $y = -2$ and $y = x$ have been drawn.
   Use the graphs to find approximate solutions to the following equations:
   (a) $x^2 - 2x - 3 = -2$
   (b) $x^2 - 2x - 3 = x$
   (c) $x^2 - 2x - 3 = 0$
   (d) $x^2 - 2x - 1 = 0$



In questions **2** to **4**, use a scale of 2 cm to 1 unit for $x$ and 1 cm to 1 unit for $y$.

**2.** Draw the graphs of the functions $y = x^2 - 2x$ and $y = x + 1$ for $-1 \leqslant x \leqslant 4$. Hence find approximate solutions of the equation $x^2 - 2x = x + 1$.

**3.** Draw the graphs of the functions $y = x^2 - 3x + 5$ and $y = x + 3$ for $-1 \leqslant x \leqslant 5$. Hence find approximate solutions of the equation $x^2 - 3x + 5 = x + 3$.

**4.** Draw the graphs of the functions $y = 6x - x^2$ and $y = 2x + 1$ for $0 \leqslant x \leqslant 5$. Hence find approximate solutions of the equation $6x - x^2 = 2x + 1$.

In questions **5** to **9**, do *not* draw any graphs.

**5.** Assuming the graph of $y = x^2 - 5x$ has been drawn, find the equation of the line which should be drawn to solve the equations:
(a) $x^2 - 5x = 3$ (b) $x^2 - 5x = -2$
(c) $x^2 - 5x = x + 4$ (d) $x^2 - 6x = 0$
(e) $x^2 - 5x - 6 = 0$

**6.** Assuming the graph of $y = x^2 + x + 1$ has been drawn, find the equation of the line which should be drawn to solve the equations:
(a) $x^2 + x + 1 = 6$ (b) $x^2 + x + 1 = 0$
(c) $x^2 + x - 3 = 0$ (d) $x^2 - x + 1 = 0$
(e) $x^2 - x - 3 = 0$

**7.** Assuming the graph of $y = 6x - x^2$ has been drawn, find the equation of the line which should be drawn to solve the equations:
(a) $4 + 6x - x^2 = 0$ (b) $4x - x^2 = 0$
(c) $2 + 5x - x^2 = 0$ (d) $x^2 - 6x = 3$
(e) $x^2 - 6x = -2$

**8.** Assuming the graph of $y = x + \frac{4}{x}$ has been drawn, find the equation of the line which should be drawn to solve the equations:
(a) $x + \frac{4}{x} - 5 = 0$ (b) $\frac{4}{x} - x = 0$
(c) $x + \frac{4}{x} = 0{\cdot}2$ (d) $2x + \frac{4}{x} - 3 = 0$
(e) $x^2 + 4 = 3x$

**9.** Assuming the graph of $y = x^2 - 8x - 7$ has been drawn, find the equation of the line which should be drawn to solve the equations:
(a) $x = 8 + \frac{7}{x}$ (b) $2x^2 = 16x + 9$
(c) $x^2 = 7$ (d) $x = \frac{4}{x - 8}$
(e) $2x - 5 = \frac{14}{x}$.

For questions **10** to **14**, use scales of 2 cm to 1 unit for $x$ and 1 cm to 1 unit for $y$.

**10.** Draw the graph of $y = x^2 - 2x + 2$ for $-2 \leqslant x \leqslant 4$. By drawing other graphs, solve the equations:
(a) $x^2 - 2x + 2 = 8$
(b) $x^2 - 2x + 2 = 5 - x$
(c) $x^2 - 2x - 5 = 0$

**11.** Draw the graph of $y = x^2 - 7x$ for $0 \leqslant x \leqslant 7$. Draw suitable straight lines to solve the equations:
(a) $x^2 - 7x + 9 = 0$
(b) $x^2 - 5x + 1 = 0$

**12.** Draw the graph of $y = x^2 + 4x + 5$ for $-6 \leqslant x \leqslant 1$. Draw suitable straight lines to find approximate solutions of the equations:
(a) $x^2 + 3x - 1 = 0$ (b) $x^2 + 5x + 2 = 0$

**13.** Draw the graph of $y = 2x^2 + 3x - 9$ for $-3 \leqslant x \leqslant 2$. Draw suitable straight lines to find approximate solutions of the equations:
(a) $2x^2 + 3x - 4 = 0$
(b) $2x^2 + 2x - 9 = 1$

**14.** Draw the graph of $y = 2 + 3x - 2x^2$ for $-2 \leqslant x \leqslant 4$.
(a) Draw suitable straight lines to find approximate solutions of the equations:
(i) $2 + 4x - 2x^2 = 0$
(ii) $2x^2 - 3x - 2 = 0$
(b) Find the range of values of $x$ for which $2 + 3x - 2x^2 \geqslant -5$.

**15.** Draw the graph of $y = \dfrac{18}{x}$ for $1 \leqslant x \leqslant 10$,
using scales of 1 cm to one unit on both axes. Use the graph to solve approximately:
(a) $\dfrac{18}{x} = x + 2$ (b) $\dfrac{18}{x} + x = 10$
(c) $x^2 = 18$

**16.** Draw the graph of $y = \frac{1}{2}x^2 - 6$ for $-4 \leqslant x \leqslant 4$, taking 2 cm to 1 unit on each axis.
(a) Use your graph to solve approximately the equation $\frac{1}{2}x^2 - 6 = 1$.
(b) Using tables or a calculator confirm that your solutions are approximately $\pm\sqrt{14}$ and explain why this is so.
(c) Use your graph to find the square roots of 8.

**17.** Draw the graph of $y = 6 - 2x - \frac{1}{2}x^3$ for $x = \pm 2, \pm 1\frac{1}{2}, \pm 1, \pm\frac{1}{2}, 0$.
Take 4 cm to 1 unit for $x$ and 1 cm to 1 unit for $y$.
Use your graph to find approximate solutions of the equations:
(a) $\frac{1}{2}x^3 + 2x - 6 = 0$ (b) $x - \frac{1}{2}x^3 = 0$
Using tables confirm that two of the solutions to the equation in part (b) are $\pm\sqrt{2}$ and explain why this is so.

**18.** Draw the graph of $y = x + \frac{12}{x} - 5$ for $x = 1$, $1\frac{1}{2}$, 2, 3, 4, 5, 6, 7, 8, taking 2 cm to 1 unit on each axis.
(a) From your graph find the range of values of $x$ for which $x + \frac{12}{x} \leqslant 9$
(b) Find an approximate solution of the equation $2x - \frac{12}{x} - 12 = 0$.

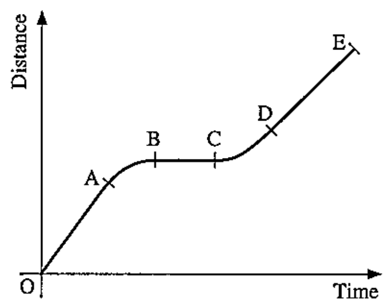
**19.** Draw the graph of $y = 2^x$ for $-4 \leqslant x \leqslant 4$, taking 2 cm to one unit for $x$ and 1 cm to one unit for $y$. Find approximate solutions to the equations:
(a) $2^x = 6$ (b) $2^x = 3x$ (c) $x2^x = 1$
Find also the approximate value of $2^{2.5}$.

**20.** Draw the graph of $y = \frac{1}{x}$ for $-4 \leqslant x \leqslant 4$ taking 2 cm to one unit on each axis. Find approximate solutions to the equations:
(a) $\frac{1}{x} = x + 1$
(b) $2x^2 - x - 1 = 0$

## 7.8 Distance–time graphs

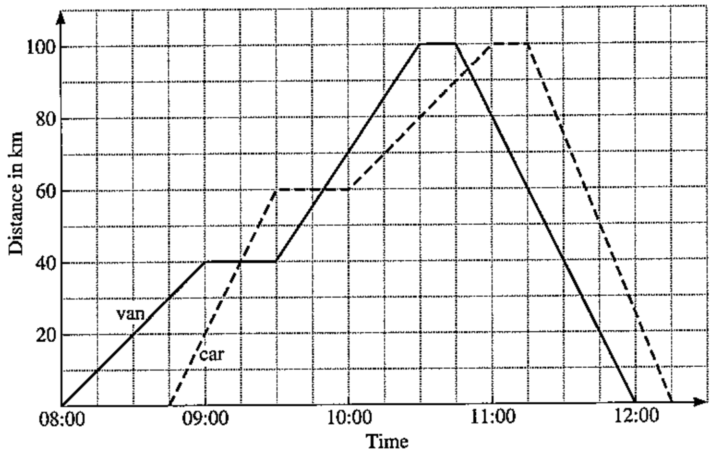
When a distance–time graph is drawn the *gradient* of the graph gives the *speed* of the object.

From O to A : constant speed
A to B : speed goes down to zero
B to C : at rest
C to D : accelerates
D to E : constant speed (not as fast as O to A)

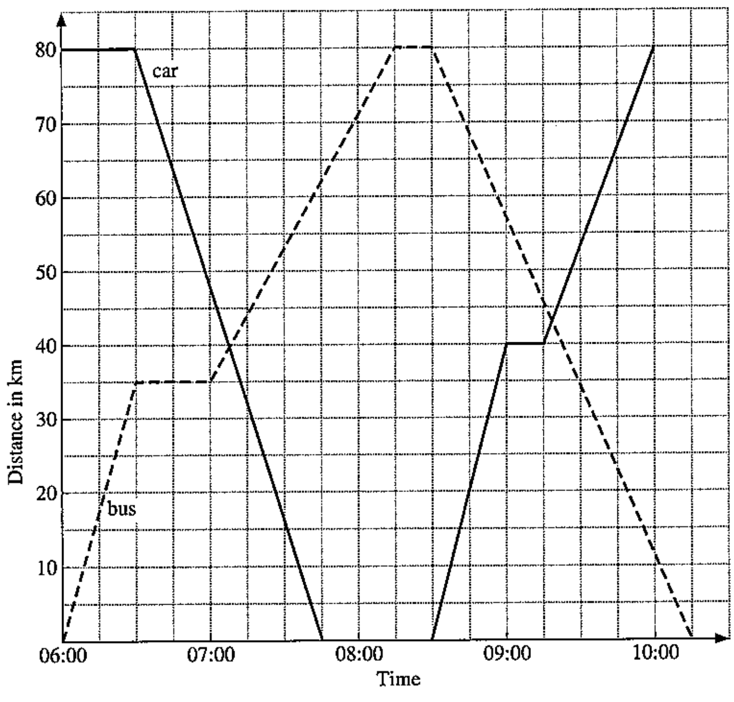


## *Exercise 11*

1. The graph shows the journeys made by a van and a car starting at York, travelling to Durham and returning to York.
   (a) For how long was the van stationary during the journey?
   (b) At what time did the car first overtake the van?
   (c) At what speed was the van travelling between 09:30 and 10:00?
   (d) What was the greatest speed attained by the car during the entire journey?
   (e) What was the average speed of the car over its entire journey?




2. The graph shows the journeys of a bus and a car along the same road. The bus goes from Leeds to Darlington and back to Leeds. The car goes from Darlington to Leeds and back to Darlington.
   (a) When did the bus and the car meet for the second time?
   (b) At what speed did the car travel from Darlington to Leeds?
   (c) What was the average speed of the bus over its entire journey?
   (d) Approximately how far apart were the bus and the car at 09:45?
   (e) What was the greatest speed attained by the car during its entire journey?



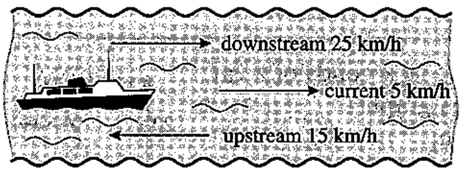

In questions **3**, **4**, **5** draw a travel graph to illustrate the journey described. Draw axes with the same scales as in question **2**.

3. Mrs Chuong leaves home at 08:00 and drives at a speed of 50 km/h. After $\frac{1}{2}$ hour she reduces her speed to 40 km/h and continues at this speed until 09:30. She stops from 09:30 until 10:00 and then returns home at a speed of 60 km/h.
   Use the graph to find the approximate time at which she arrives home.

**4.** Mr Coe leaves home at 09:00 and drives at a speed of 20 km/h. After $\frac{3}{4}$ hour he increases his speed to 45 km/h and continues at this speed until 10:45. He stops from 10:45 until 11:30 and then returns home at a speed of 50 km/h.
Draw a graph and use it to find the approximate time at which he arrives home.

**5.** At 10:00 Akram leaves home and cycles to his grandparents' house which is 70 km away. He cycles at a speed of 20 km/h until 11:15, at which time he stops for $\frac{1}{2}$ hour. He then completes the journey at a speed of 30 km/h. At 11:45 Akram's sister, Hameeda, leaves home and drives her car at 60 km/h. Hameeda also goes to her grandparents' house and uses the same road as Akram.
At approximately what time does Hameeda overtake Akram?

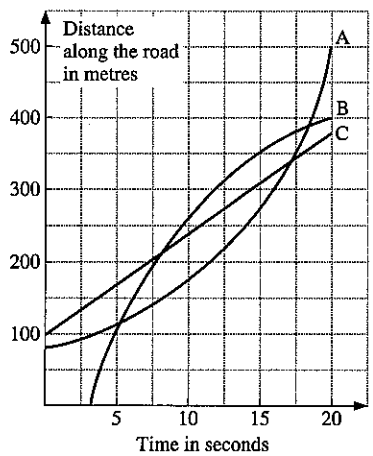
**6.** A boat can travel at a speed of 20 km/h in still water. The current in a river flows at 5 km/h so that downstream the boat travels at 25 km/h and upstream it travels at only 15 km/h.



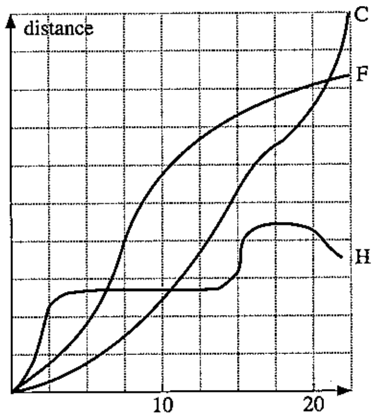

The boat has only enough fuel to last 3 hours.
The boat leaves its base and travels downstream.
Draw a distance–time graph and draw lines to indicate the outward and return journeys.
After what time must the boat turn round so that it can get back to base without running out of fuel?

**7.** The boat in question **6** sails in a river where the current is 10 km/h and it has fuel for four hours. At what time must the boat turn round this time if it is not to run out of fuel?

**8.** The graph shows the motion of three cars A, B and C along the same road.
Answer the following questions giving estimates where necessary.
(a) Which car is in front after
(i) 10 s, (ii) 20 s?
(b) When is B in the front?
(c) When are B and C going at the same speed?
(d) When are A and C going at the same speed?
(e) Which car is going fastest after 5 s?
(f) Which car starts slowly and then goes faster and faster?

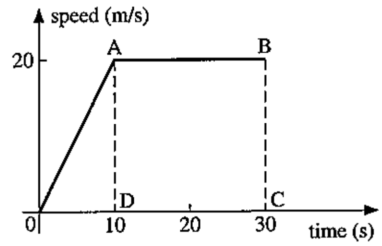


9. Three girls Hanna, Fateema and Carine took part in an egg and spoon race. Describe what happened, giving as many details as possible.




## 7.9 Speed–time graphs

The diagram is the speed–time graph of the first 30 seconds of a car journey. Two quantities are obtained from such graphs:
(a) acceleration = gradient of speed–time graph,
(b) distance travelled = area under graph.




In this example,

(a) The gradient of line OA $= \frac{20}{10} = 2$

$\therefore$ The acceleration in the first 10 seconds is $2\,\text{m/s}^2$.

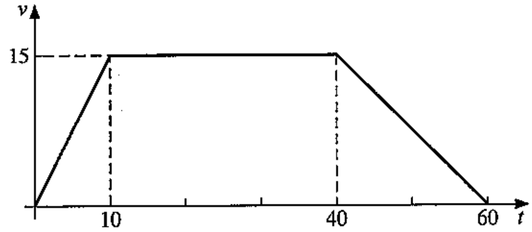
(b) The distance travelled in the first 30 seconds is given by the area of OAD plus the area of ABCD.

$$\text{Distance} = (\tfrac{1}{2} \times 10 \times 20) + (20 \times 20)$$
$$= 500\,\text{m}$$

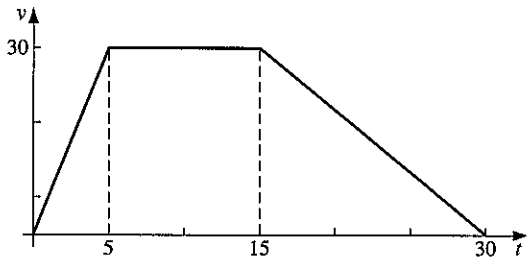
### *Exercise 12*

On the graphs in this exercise speeds are in m/s and all times are in seconds.

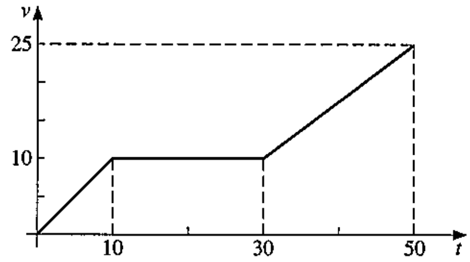
1. Find:
   (a) the acceleration when $t = 4$,
   (b) the total distance travelled,
   (c) the average speed for the whole journey.



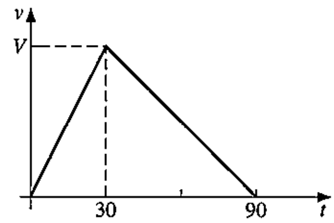
**2.** Find:
(a) the total distance travelled,
(b) the average speed for the whole journey,
(c) the distance travelled in the first 10 seconds,
(d) the acceleration when $t = 20$.



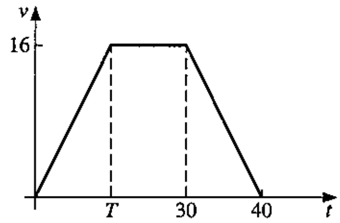

**3** Find:
(a) the total distance travelled,
(b) the distance travelled in the first 40 seconds,
(c) the acceleration when $t = 15$.



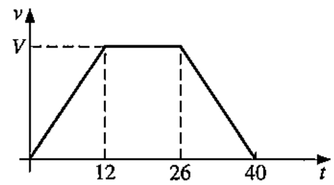

**4.** Find:
(a) $V$ if the total distance travelled is 900 m,
(b) the distance travelled in the first 60 seconds.



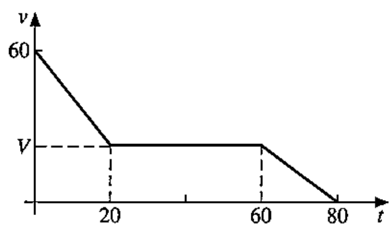

**5.** Find:
(a) $T$ if the initial acceleration is $2\,\text{m/s}^2$,
(b) the total distance travelled,
(c) the average speed for the whole journey.



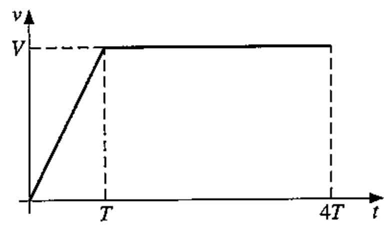

**6.** Given that the total distance travelled = 810 m, find:
(a) the value of $V$,
(b) the rate of change of the speed when $t = 30$,
(c) the time taken to travel the first 420 m of the journey.



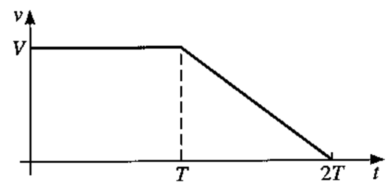

**7.** Given that the total distance travelled is 1·5 km, find:
(a) the value of $V$,
(b) the rate of deceleration after 10 seconds.



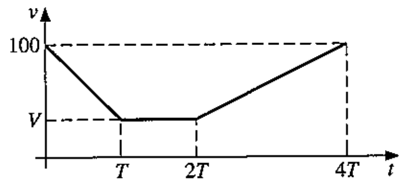
8. Given that the total distance travelled is 1·4 km, and the acceleration is $4\,m/s^2$ for the first $T$ seconds, find:
   (a) the value of $V$,
   (b) the value of $T$.




9. Given that the average speed for the whole journey is 37·5 m/s and that the deceleration between $T$ and $2T$ is $2{\cdot}5\,m/s^2$, find:
   (a) the value of $V$,
   (b) the value of $T$.




10. Given that the total distance travelled is 4 km and that the initial deceleration is $4\,m/s^2$, find:
    (a) the value of $V$,
    (b) the value of $T$.




## *Exercise 13*

Sketch a speed–time graph for each question.
All accelerations are taken to be uniform.

1. A car accelerated from 0 to 50 m/s in 9s. How far did it travel in this time?

2. A motor cycle accelerated from 10 m/s to 30 m/s in 6s. How far did it travel in this time?

3. A train slowed down from 50 km/h to 10 km/h in 2 minutes. How far did it travel in this time?

4. When taking off, an aircraft accelerates from 0 to 100 m/s in a distance of 500 m. How long did it take to travel this distance?

5. An earthworm accelerates from a speed of 0·01 m/s to 0·02 m/s over a distance of 0·9 m. How long did it take?

6. A car travelling at 60 km/h is stopped in 6 seconds. How far does it travel in this time? [Hint: Change 6 seconds into hours.]

7. A car accelerates from 15 km/h to 60 km/h in 3 seconds. How far does it travel in this time?

8. At lift-off a rocket accelerates from 0 to 1000 km/h in just 10s. How far does it travel in this time?

9. A coach accelerated from 0 to 60 km/h in 30s. How many metres did it travel in this time?

**10.** Hamad was driving a car at 30 m/s when he saw an obstacle 45 m in front of him. It took a reaction time of 0·3 seconds before he could press the brakes and a further 2·5 seconds to stop the car. Did he hit the obstacle?

**11.** An aircraft is cruising at a speed of 200 m/s. When it lands it must be travelling at a speed of 50 m/s. In the air it can slow down at a rate of 0·2 m/s$^2$. On the ground it slows down at a rate of 2 m/s$^2$. Draw a velocity–time graph for the aircraft as it reduces it speed from 200 m/s to 50 m/s and then to 0 m/s.
How far does it travel in this time?

**12.** The speed of a train is measured at regular intervals of time from $t = 0$ to $t = 60$s, as shown below.

| $t$ s | 0 | 10 | 20 | 30 | 40 | 50 | 60 |
|---|---|---|---|---|---|---|---|
| $v$ m/s | 0 | 10 | 16 | 19·7 | 22·2 | 23·8 | 24·7 |

Draw a speed–time graph to illustrate the motion. Plot $t$ on the horizontal axis with a scale of 1 cm to 5s and plot $v$ on the vertical axis with a scale of 2 cm to 5 m/s.
Use the graph to estimate:
(a) the acceleration at $t = 10$,
(b) the distance travelled by the train from $t = 30$ to $t = 60$.

[An approximate value for the area under a curve can be found by splitting the area into several trapeziums.]

**13.** The speed of a car is measured at regular intervals of time from $t = 0$ to $t = 60$s, as shown below.

| $t$ s | 0 | 10 | 20 | 30 | 40 | 50 | 60 |
|---|---|---|---|---|---|---|---|
| $v$ m/s | 0 | 1·3 | 3·2 | 6 | 10·1 | 16·5 | 30 |

Draw a speed–time graph using the same scales as in question **11**.
Use the graph to estimate:
(a) the acceleration at $t = 30$.
(b) the distance travelled by the car from $t = 20$ to $t = 50$.

## Revision exercise 7A

**1.** Find the equation of the straight line satisfied by the following points:

(a)

| $x$ | 2 | 7 | 10 |
|---|---|---|---|
| $y$ | −5 | 0 | 3 |

(b)

| $x$ | 1 | 2 | 3 |
|---|---|---|---|
| $y$ | 7 | 9 | 11 |

(c)

| $x$ | 1 | 2 | 3 |
|---|---|---|---|
| $y$ | 8 | 6 | 4 |

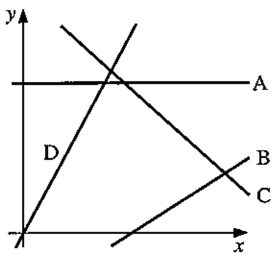
(d)

| $x$ | 3 | 4 | 5 |
|---|---|---|---|
| $y$ | 2 | $2\frac{1}{2}$ | 3 |

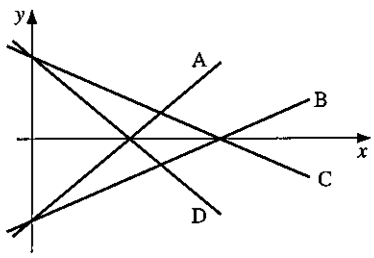
2. Find the gradient of the line joining each pair of points.
   (a) (3, 3)(5, 7) (b) (3, −1)(7, 3)
   (c) (−1, 4)(1, −3) (d) (2, 4)(−3, 4)
   (e) (0·5, −3)(0·4, −4)

3. Find the gradient and the intercept on the $y$-axis for the following lines. Draw a *sketch* graph of each line.
   (a) $y = 2x - 7$ (b) $y = 5 - 4x$
   (c) $2y = x + 8$ (d) $2y = 10 - x$
   (e) $y + 2x = 12$ (f) $2x + 3y = 24$

4. In the diagram, the equations of the lines are $y = 3x$, $y = 6$, $y = 10 - x$ and $y = \frac{1}{2}x - 3$.
   Find the equation corresponding to each line.

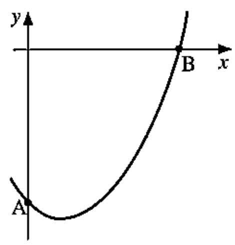



5. In the diagram, the equations of the lines are $2y = x - 8$, $2y + x = 8$, $4y = 3x - 16$ and $4y + 3x = 16$.
   Find the equation corresponding to each line.




6. Find the equations of the lines which pass through the following pairs of points:
   (a) (2, 1)(4, 5) (b) (0, 4)(−1, 1)
   (c) (2, 8)(−2, 12) (d) (0, 7)(−3, 7)

7. The sketch represents a section of the curve $y = x^2 - 2x - 8$.
   Calculate:
   (a) the coordinates of A and of B,
   (b) the gradient of the line AB,
   (c) the equation of the straight line AB.




8. Find the area of the triangle formed by the intersection of the lines $y = x$, $x + y = 10$ and $x = 0$.

9. Draw the graph of $y = 7 - 3x - 2x^2$ for $-4 \leqslant x \leqslant 2$.
   Find the gradient of the tangent to the curve at the point where the curve cuts the $y$-axis.

10. Draw the graph of $y = \dfrac{4000}{x} + 3x$ for $10 \leqslant x \leqslant 80$. Find the minimum value of $y$.

**11.** Draw the graph of $y = \frac{1}{x} + 2^x$ for $x = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$, 1, $1\frac{1}{2}$, 2, 3.

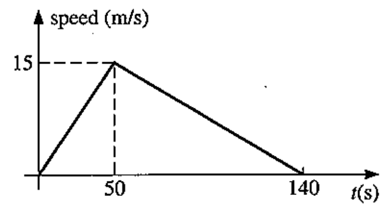
**12.** Assuming that the graph of $y = 4 - x^2$ has been drawn, find the equation of the straight line which should be drawn in order to solve the following equations:
(a) $4 - 3x - x^2 = 0$ (b) $\frac{1}{2}(4 - x^2) = 0$
(c) $x^2 - x + 7 = 0$ (d) $\frac{4}{x} - x = 5$

**13.** Draw the graph of $y = 5 - x^2$ for $-3 \leqslant x \leqslant 3$, taking 2 cm to one unit for $x$ and 1 cm to one unit for $y$.
Use the graph to find:
(a) approximate solutions to the equation $4 - x - x^2 = 0$,
(b) the square roots of 5,
(c) the square roots of 7.

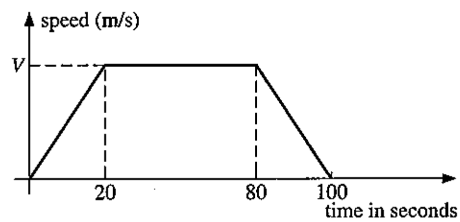
**14.** Draw the graph of $y = \frac{5}{x} + 2x - 3$, for $\frac{1}{2} \leqslant x \leqslant 7$, taking 2 cm to one unit for $x$ and 1 cm to one unit for $y$.
Use the graph to find:
(a) approximate solutions to the equation $2x^2 - 10x + 7 = 0$,
(b) the range of values of $x$ for which
$\frac{5}{x} + 2x - 3 < 6$.
(c) the minimum value of $y$.

**15.** Draw the graph of $y = 4^x$ for $-2 \leqslant x \leqslant 2$.
Use the graph to find:
(a) the approximate value of $4^{1\cdot 6}$,
(b) the approximate value of $4^{-\frac{1}{3}}$,
(c) the gradient of the curve at $x = 0$
(d) an approximate solution to the equation $4^x = 10$.

**16.** The diagram is the speed–time graph of a bus.
Calculate:
(a) the acceleration during the first 50 seconds,
(b) the total distance travelled,
(c) how long it takes before it is moving at 12 m/s for the first time.




**17.** The diagram is the speed–time graph of a car.
Given that the total distance travelled is 2·4 km, calculate:
(a) the value of the maximum speed $V$,
(b) the distance travelled in the first 30 seconds of the motion.



## Examination exercise 7B

**1.** Answer the whole of this question on a sheet of graph paper.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $y$ | 0 | $-6$ | $-8$ | $-6$ | 0 | 10 |

(a) Using a scale of 2 cm to represent 1 unit on the $x$-axis, and a scale of 2 cm to represent 4 units on the $y$-axis, plot the points given in the table above.
Join the points with a smooth curve.

(b) (i) On the same grid, draw the line with gradient 2 through the point (3, 0). Label it L.
(ii) Write down the equation of the line L.
(iii) Write down the coordinates of the two points at which the line L meets the curve.

(c) Draw the tangent to the curve at the point (3, $-6$) and use it to find an estimate of the gradient of the curve at that point.

(d) The equation of the curve is $y = ax^2 + bx$, where $a$ and $b$ are integers.
Using some of the values from the table above, calculate $a$ and $b$. N 97 4

**2.** Answer the whole of this question on a sheet of graph paper.
The tables below give values of $f(x)$ and $g(x)$, correct to 1 decimal place.

$$f(x) = \frac{12}{x}$$

| $x$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 6 | 4 | 3 | $p$ | $q$ | 1·7 | $r$ | 1·3 |

$$g(x) = (x - 8)(2 - x)$$

| $x$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| $g(x)$ | 0 | $s$ | 8 | $t$ | 8 | 5 | 0 | $u$ |

(a) Calculate the values of $p$, $q$, $r$, $s$, $t$ and $u$.

(b) Using a scale of 2 cm to represent 1 unit, draw an $x$-axis for $0 \leqslant x \leqslant 9$ and using a scale of 1 cm to represent 1 unit, draw a $y$-axis for $-8 \leqslant y \leqslant 10$.
Draw the graphs of $y = f(x)$ and $y = g(x)$ for $2 \leqslant x \leqslant 9$ on the same grid.

(c) (i) Show that the equation which is satisfied by the points of intersection of the graphs can be simplified to:

$$x^3 - 10x^2 + 16x + 12 = 0$$

(ii) Write down the two solutions to this equation which can be found from your graphs. Give your answers correct to 1 decimal place.

(d) Draw the tangent to $y = g(x)$ at the point (7, 5).
Use this to estimate the gradient of the curve $y = g(x)$ at this point. J 96 4

**3.** Answer the whole of this question on a sheet of graph paper.

A table of values for $y = \frac{4}{x^2} + x$ is given below.

(The values of $y$ are correct to one decimal place.)

| $x$ | $-2$ | $-1{\cdot}5$ | $-1{\cdot}2$ | $-1$ | $-0{\cdot}8$ | $1$ | $1{\cdot}5$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | $-1$ | $0{\cdot}3$ | $1{\cdot}6$ | $l$ | $5{\cdot}5$ | $m$ | $3{\cdot}3$ | $3$ | $n$ | $4{\cdot}3$ |

(a) Calculate the values of $l$, $m$ and $n$.

(b) Using a scale of 2 cm to represent 1 unit on both axes, draw the graph of

$y = \frac{4}{x^2} + x$ for $-2 \leqslant x \leqslant -0{\cdot}8$ and

$1 \leqslant x \leqslant 4$.

(c) Use your graph to solve:

(i) $\frac{4}{x^2} + x = 0$ (ii) $\frac{4}{x^2} + x = 4$

(d) By drawing a suitable tangent to the curve, estimate the gradient of the curve when $x = 1{\cdot}5$. N 98 4

**4.** Answer the whole of this question on a sheet of graph paper.

A table of values for $y = x(x+2)(x-3)$ is given below.

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|---|---|
| $y$ | $a$ | $0$ | $b$ | $0$ | $-6$ | $c$ | $0$ | $24$ |

(a) Calculate the values of $a$, $b$ and $c$.

(b) Using a scale of 2 cm to represent 1 unit, draw an $x$-axis for $-3 \leqslant x \leqslant 4$ and using a scale of 2 cm to represent 5 units, draw a $y$-axis for $-20 \leqslant y \leqslant 25$.

Draw the graph of $y = x(x+2)(x-3)$.

(c) Use your graph to solve:

(i) $x(x+2)(x-3) = 10$

(ii) $x(x+2)(x-3) + 15 = 0$

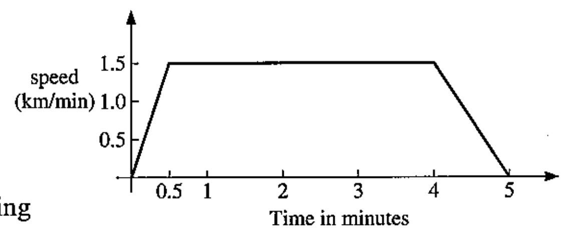
(d) Draw the line $y = 2x - 6$ on your graph.

(e) The graphs meet when $x(x+2)(x-3) = 2x - 6$.

(i) Show that this equation can be written as $x^3 - x^2 - 8x + 6 = 0$

(ii) Write down the solutions of this equation. J 98 4

**5.** The graph shows the speed of a car during a five-minute journey.




(a) For how long does the car travel at a steady speed?

(b) What is the acceleration of the car during the first half minute?

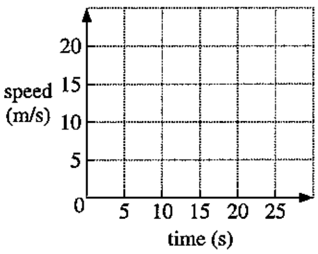
(c) Calculate the distance travelled by the car during

(i) the first half minute of the journey,

(ii) the whole journey. N 96 2

**6.** A car accelerates steadily from rest to a speed of 20 metres per second in 15 seconds.

(a) Draw the speed–time graph on a copy of the grid below.




(b) Calculate the acceleration, in metres per second per second.

(c) Calculate the distance travelled in these 15 seconds. N 98 2

**7.** Answer the whole of this question on a sheet of graph paper.

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $f(t)$ | 0 | 25 | 37·5 | 43·8 | 46·9 | 48·4 | 49·2 | 49·6 |

(a) Using a scale of 2 cm to represent 1 unit on the horizontal $t$-axis and 2 cm to represent 10 units on the $y$-axis, draw axes for $0 \leqslant t \leqslant 7$ and $0 \leqslant y \leqslant 60$.

(b) $f(t) = 50(1 - 2^{-t})$.

(i) Calculate the value of $f(8)$ and the value of $f(9)$.

(ii) Estimate the value of $f(t)$ when $t$ is large.

(c) (i) Draw the tangent to $y = f(t)$ at $t = 2$ and use it to calculate an estimate of the gradient of the curve at this point.

(ii) The function $f(t)$ represents the speed of a particle at time $t$. Write down what quantity the gradient gives.

(d) (i) On the same grid, draw $y = g(t)$ where $g(t) = 6t + 10$, for $0 \leqslant t \leqslant 7$.

(ii) Write down the range of values for $t$ where $f(t) > g(t)$.

(iii) The function $g(t)$ represents the speed of a second particle at time $t$. State whether the first or second particle travels the greater distance for $0 \leqslant t \leqslant 7$.
You **must** give a reason for your answer. N 03 4